

Partial Wave Analysis of Pion Electroproduction Data

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ABSTRACT

The high duty factor accelerator at CEBAF will provide the possibility to conduct high statistics single pion electroproduction experiments over a large Q^2 and W range. One may take advantage of the large acceptance of the CLAS detector to undertake experiments, the data of which can be used to perform a model and energy-independent partial wave analysis.

Introduction

The measurement of pion electroproduction in the nucleon resonance region provides information on the electromagnetic transition amplitudes of the nucleon. To obtain this information several high statistic experiments are planned with the CLAS detector at CEBAF^[1]. The CLAS detector is a magnetic multi-gap spectrometer based on a large iron-free toroid with six superconducting coils. The particle detection system consists of drift chambers to determine the tracks of charged particles, scintillation counters for the trigger and for time-of-flight, and Cerenkov and shower counters to identify electrons and detect photons. The expected momentum resolution is 0.2% (sigma) for momenta up to 4 GeV/c. The detector should be able to operate at a maximum luminosity of $10^{34} \text{ cm}^{-2} \text{ sec}^{-1}$.

Previous measurements^[2] of the $p(e, e'p)\pi^0$ and $p(e, e'\pi^+)n$ reactions have been limited by low statistical precision, particularly in the high invariant mass W and Q^2 region, and by the limited geometrical acceptance. In addition, these measurements covered a small angular range in the azimuthal angle φ and polar angle θ in the hadronic center of mass. Due to these limitations, phenomenological fitting procedures have been used to obtain information on transition amplitudes (γNN^* vertex). The analyses were made in terms of a model in which the amplitude consisted of three separate contributions: (1) the Born terms, (2) resonances described by the Breit-Wigner formula for which the positions and widths were taken from pion-nucleon scattering data, but whose amplitudes were adjustable parameters, and (3) additional nonresonant background terms. This energy dependent analysis generally finds only those resonances which have been build into the parametrization. Uncertainties in the basic resonance parameters (masses and widths) and any error in the analysis of pion-nucleon scattering can propagate into the electroproduction analysis. There is a significant difference between *assuming* a resonance is electroproduced in an energy-dependent analysis and *discovering* it in an energy-independent analysis. The properties, and occasionally even the detection, of resonances in non-leading partial waves depend on the details of the parametrization. For example, the choice of Breit-Wigner phases (interference) can eliminate physical solutions.

This paper discusses the general procedure^[3] to obtain differential cross sections and to disentangle the various resonant partial waves. As a specific example, the events are corrected for configuration-dependent inefficiencies and acceptance losses in the CLAS detector. We show that the CLAS detector has sufficient coverage for the partial wave analysis. In section I the method of data analysis for exclusive reactions is described. In section II we demonstrate the feasibility of the proposed method using Monte Carlo simulation.

I. Method of Analysis

I.1 Correction for detection inefficiencies

A fraction of produced events that would have been accepted by the geometry of the CLAS detector is not observed because of various detection inefficiencies. These inefficiencies depend on event geometry and kinematics and may be corrected for event by event. The correction is usually performed by introducing for every event i an individual weight w_i which is calculated as a product of all weight factors $w_i^{(\alpha)}$ arising from inefficiencies of type α . Here are a few examples:

- 1) *Decaying pions.* Events with secondary pions that decayed along their path through the drift chambers may fail a reconstruction. This loss may be accounted for by the weight:

$$w^{(d)} = 1 + \frac{m_\pi}{c\tau p_\pi} L_d$$

where L_d is that part of the trajectory along which a decay will produce a reconstruction program failure, p_π is the momentum of the pion, and $c\tau = 780$ cm.

- 2) *Secondary interactions.* The weight $w^{(int)}$ due to secondary particle interactions in the drift chamber material can be calculated from the path length of the secondaries (that part of the trajectory which will produce a reconstruction program failure) and their total cross sections.
- 3) *Cerenkov and shower counter inefficiencies* depend also on the event geometry and kinematics, and therefore may influence the shape of the differential cross section.
- 4) *Trigger inefficiencies*, and so on.

I.2 General procedure to obtain differential cross sections

The kinematics of an event can be described completely by a set of independent kinematical variables x_K which are invariant under event rotation or translation in the lab frame. The differential production cross section may be defined as an intensity distribution in these variables: $I_{prod}(x_K)$. At fixed beam energy we chose as kinematical variables for the reaction

$$e + p \rightarrow e' + N^* \rightarrow e' + N + \pi \quad (1)$$

the four quantities W, Q^2, θ, φ , where W is the N^* invariant mass, Q^2 the squared four momentum transfer between e and e' , and where θ, φ define the π direction in the hadronic center of mass. The angle θ is the angle between the decay pion momentum and the direction of virtual photon. The angle φ is the azimuthal

angle of the pion relative to the electron scattering plane (e, e'). To specify the configuration of an event completely, one needs in addition to x_K a set of laboratory coordinates x_L giving its position and orientation in the laboratory. Thus the distribution of event configurations in the laboratory is described by

$$I'_{prod}(x_K, x_L) = I_{prod}(x_K)P(x_L) \quad (2)$$

where $P(x_L)$ is the spatial distribution of produced events, normalized to unity. Only part of the distribution $I'_{prod}(x_K, x_L)$ is accepted by the geometry of the detector: we call it the "accepted" distribution $I'_{acc}(x_K, x_L)$. Between I'_{prod} and I'_{acc} one has the relation

$$I'_{acc}(x_K, x_L) = A'(x_K, x_L)I'_{prod}(x_K, x_L) \quad (3)$$

where the acceptance propability $A'(x_K, x_L)$ takes the values 1 or 0 depending on whether the event (x_K, x_L) lies inside the geometry of the apparatus or not. Integrating eq.(3) over x_L one obtains with eq.(2)

$$I_{acc}(x_K) = \int I'_{acc}(x_K, x_L)dx_L = A(x_K)I_{prod}(x_K) \quad (4)$$

where the acceptance function $A(x_K)$ is defined as

$$A(x_K) = \int A'(x_K, x_L)P(x_L)dx_L \quad (5)$$

Owing to the configuration-dependent detection inefficiencies discussed in section I.1, $I'_{acc}(x_K, x_L)$ is not identical to the distribution of actually observed events $I'_{obs}(x_K, x_L)$ but differs by weight factor $w(x_K, x_L)$

$$I'_{acc}(x_K, x_L) = w(x_K, x_L)I'_{obs}(x_K, x_L) \quad (6)$$

In practice, $I_{acc}(x_K)$ is obtained from $I'_{obs}(x_K, x_L)$ by attaching a weight $w_i = w[x_K^{(i)}, x_L^{(i)}]$ to each individual event i , as already mentioned in section I.1.

Starting from the "accepted" distribution $I_{acc}(x_K)$, which is identical to the weighted observed distribution, relation (4) will give directly the distribution of produced events in the domain of variables x_K , where $A(x_K)$ is greater then zero. In order to obtain $I_{prod}(x_K)$ also in the region where the acceptance is zero, one has to parametrize $I_{prod}(x_K)$ and to determine the parameters in the observed domain of variables.

The acceptance function $A(x_K)$ as defined by relation (5) may be obtained by a Monte-Carlo calculation. For fixed W and Q^2 values, events are generated at random in the variables $\cos\theta, \phi$ and transformed into the lab system according to the distribution $P(x_L)$, which can include effects of the measured beam distribution

in the target. The acceptance $A(x_K)$ is then simply that fraction of all events generated with x_K which lies within the accepted geometry region as defined by the drift chambers and the arrangement of Cerenkov and trigger counters. To avoid edge effects, the acceptance windows introduced for generated as well as for observed events should be chosen somewhat smaller than those imposed by the detector geometry. For each rather narrow intervals $\Delta W \Delta Q^2$ the average acceptance has to be determined in each $\Delta \Omega$ bin, for example: $\Delta \Omega = \Delta \cos \theta \Delta \phi = (\frac{2}{40})(\frac{2\pi}{36})$, by generating at least 144,000 events (100 events/bin).

I.3 Fits to the $p\pi$ angular distribution

Due to parity conservation in the production process (1), the most general $p\pi$ angular distribution with $(\cos \theta, \phi) = \Omega$ defined in the center of mass reference frame where the y -axis is perpendicular to the electron scattering plane and z -axis is along the virtual photon direction, can be written in terms of spherical harmonics $Y_L^M(\Omega)$ as follows:

$$I_{prod}(W, Q^2, \Omega) = \sum_{L,M \geq 0} t_L^M \text{Re} Y_L^M(\Omega) \quad (7)$$

Here the real parameters t_L^M are functions of W and Q^2 . Since the fits to the $p\pi$ angular distribution are carried out in rather narrow intervals $\Delta W \Delta Q^2$, we consider the t_L^M as constants within a W, Q^2 bin and omit the variables W and Q^2 from the arguments of I_{prod} . The distribution $I_{prod}(\Omega)$ is assumed to be normalized to the number of produced events N_{prod} in $\Delta W \Delta Q^2$:

$$\int I_{prod}(\Omega) d\Omega = N_{prod} = \sqrt{4\pi} t_0^0 \quad (8)$$

The t_L^M are related to the often-used normalized spherical harmonic moments $\langle \text{Re} Y_L^M \rangle$:

$$\langle \text{Re} Y_L^M \rangle = \frac{1}{N_{prod}} \int I_{prod}(\Omega) \text{Re} Y_L^M(\Omega) d\Omega = \frac{t_L^M \epsilon_L^M}{N_{prod}} \quad (9)$$

where $\epsilon_L^M = 1$ for $M = 0$ and $\epsilon_L^M = \frac{1}{2}$ for $M \neq 0$. Throughout the rest of this paper we will simply write Y_L^M as shorthand for $\text{Re} Y_L^M$. Furthermore, for simpler notation in subsequent formulae, we will also compress L, M into a single index λ , with the convention that $\lambda = 0$ corresponds to the case $L = 0, M = 0$.

Inserting eq.(7) into eq.(4) we obtain the basic relation for all angular distribution fits:

$$I_{acc}(\Omega) = A(\Omega) \sum_{\lambda=0}^{\lambda_{max}} t_{\lambda} Y_{\lambda}(\Omega) \quad (10)$$

In the following subsections, two independent fit methods (χ^2 method and method of moments) will be described which can be employed to determine the parameters

of the $p\pi$ angular distribution. From the χ^2 or moment method discussed below, we can obtain $A(\Omega)$ for a considered W, Q^2 interval by averaging the acceptance functions, calculated for fixed W, Q^2 values, according to the distribution of observed events in that interval.

I.3.1 χ^2 method

The procedure consists of fitting the accepted distribution I_{acc} as defined by eq.(10) to the weighted observed event distribution in a grid of $\cos\theta, \phi$ bins of equal size $\Delta\Omega$. Defining w_k as the total weight of observed events in bin k ($w_k = \sum_i w_i$ as already discussed in section 1) and w_k^{exp} as the expected total weight in the same bin, the χ^2 to be minimized can be written as

$$\chi^2 = \sum_k \frac{[w_k - w_k^{exp}]^2}{\sigma_k^2} \quad (11)$$

with $w_k^{exp} = (\Delta\Omega/4\pi)A(\Omega_k) \sum_\lambda t_\lambda Y_\lambda(\Omega_k)$.

In the fits $Y_\lambda(\Omega_k)$ is evaluated at the bin center whereas for $A(\Omega_k)$ the average acceptance in the bin is used. The squared variance of the expected total weight w_k^{exp} can be shown to be

$$\sigma_k^2 = \frac{\langle w_k^2 \rangle}{\langle w_k \rangle} w_k^{exp} \quad (12)$$

where $\langle w_k \rangle$ is the mean event weight and $\langle w_k^2 \rangle$ is the mean squared weight in bin k . In case, our weights depend very weakly on $\cos\theta, \phi$, constant values $\langle w \rangle$ and $\langle w^2 \rangle$ may be evaluated from all events included in the fit, and thus can be used for all bins. The same bin size $\Delta\Omega$ should be chosen as in the acceptance calculation. Bins with $A(\Omega_k) < A(\Omega_k)_{min}$ should be excluded from the fit.

The χ^2 minimization is achieved by solving the system of equations $\delta\chi^2/\delta t_\lambda = 0$ for t_λ with standard iterative procedures. The error matrix $E(t_\lambda)$ of the fitted t_λ is obtained from the relation

$$[E(t_\mu)^{-1}]_{\lambda\lambda'} = -\frac{1}{2} \frac{\delta^2 \chi^2}{\delta t_\lambda \delta t_{\lambda'}} \quad (13)$$

I.3.2 Linear algebra method (method of moments)

Multiplying eq.(10) with $Y_{\lambda'}(\Omega)$ and integrating over Ω yields

$$\int I_{acc}(\Omega) Y_{\lambda'} d\Omega = \sum_\lambda \left\{ \int Y_{\lambda'}(\Omega) A(\Omega) Y_\lambda(\Omega) d\Omega \right\} t_\lambda \quad (14)$$

For easier notation we define the experimental moments

$$b_\lambda = \int I_{acc} Y_\lambda d\Omega \quad (15)$$

and the acceptance correlations

$$A_{\lambda\lambda'} = \int Y_{\lambda'} A Y_{\lambda} d\Omega \quad (16)$$

Thus eq.(14) can be written in simpler form:

$$b_{\lambda'} = \sum_{\lambda} A_{\lambda\lambda'} t_{\lambda} \quad (17)$$

For a set of different λ' , relation (17) defines a system of linear equations which can be solved for the t_{λ} as soon as one employs as many equations as there are t_{λ} admitted. Using more equations than unknowns yields an over-determination allowing for a real fit which will reduce the errors in t_{λ} . This fit is achieved by minimizing

$$\chi^2 = \sum_{\mu\mu'} d_{\mu} [E(b)^{-1}]_{\mu\mu'} d_{\mu'} \quad (18)$$

where $d_{\mu} = \sum_{\lambda} A_{\mu\lambda} t_{\lambda} - b_{\mu}$, and where $E(b)$ is the error matrix of the experimental moments b_{μ} . In eq.(18) we have neglected possible errors in the acceptance correlations $A_{\mu\lambda}$ which in principle can be calculated to any required precision. The minimization of the χ^2 defined in eq.(18) is still a linear problem which is solved by standard methods of matrix algebra.

In practice the b_{λ} , as defined by eq.(15), are obtained by summing over observed events:

$$b_{\lambda} = \sum_{i=1}^{N_{obs}} w_i Y_{\lambda}(\Omega_i) \quad (19)$$

where w_i is the weight of event i and N_{obs} the number of observed events included in the fit. Similarly, the acceptance correlations are calculated by summation over acceptance bins in the Monte-Carlo calculations:

$$A_{\lambda\lambda'} = \frac{4\pi}{N_{gen}} \sum_{j=1}^{N_{bin}} n_j Y_{\lambda'}(\Omega_j) Y_{\lambda}(\Omega_j) \quad (20)$$

where n_j is the number of accepted events in each of the $\Delta\cos\theta\Delta\phi$ bin, N_{gen} the number of generated events in the Monte-Carlo calculation, and $Y_{\lambda}(\Omega_j)$ the values of the spherical harmonics in the center of the bin. The error matrix of the experimental moments b_{λ} is

$$E(b)_{\mu\mu'} = \sum_{i=1}^{N_{obs}} w_i^2 Y_{\mu}(\Omega_i) Y_{\mu'}(\Omega_i) \quad (21)$$

From $E(b)$ and the acceptance correlation matrix A , one obtains the error matrix

$E(t)$ of the fitted expansion coefficients t_λ . In matrix notation we have

$$E(t) = [A^T E(b)^{-1} A]^{-1} \quad (22)$$

I.4 Absolute cross section

The fitted distribution of produced events $I_{prod}(x_K)$ as defined in section I.3 is corrected only for configuration-dependent and acceptance losses. In order to obtain an absolute cross section one has to consider in addition the luminosity and a number of losses which are independent of the event kinematics and geometry, and can therefore be accounted for by applying a constant global weight factor w_g to $I_{prod}(x_K)$. For example:

- 1) Luminosity,
- 2) Reconstruction losses (drift chamber inefficiencies, wrong left-right ambiguity solution,...),
- 3) Loss of events due to missing mass cut,
- 4) Losses due to electronics inefficiency (losses of trigger due to dead time selection,...), and so on.

Thus, we find that one produced event corresponds to a cross section of $n.nn$ nanobarn. From this value and the number of produced events in the interval $\Delta W \Delta Q^2 \Delta \Omega$ one determines the differential cross sections for reaction (1).

II. Monte Carlo Simulation

II.1 Event Simulation

We have generated 1.6 million events of the reaction $p(e, e'p)\pi^0$ in the mass W range from 1.1 to 1.7 GeV at the fixed $Q^2 = 2.0 \text{ GeV}^2$. The mass distribution consists of five resonances (Breit-Wigner forms added incoherently): $P_{33}(1232)$, $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$ and $F_{15}(1680)$ taken with the following ratios 40 : 15 : 15 : 15 : 15. For each resonance, the center of mass angular distribution with definite angular momentum l , was generated in terms of spherical harmonics $Y_l^m(\cos\theta, \varphi)$ with $m = 0$ (uniform φ distribution). However, the additional φ -dependence of the cross section σ :

$$\sigma = \sigma_T + \epsilon\sigma_L + \epsilon\sigma_{TT}\cos 2\varphi + \sqrt{\frac{\epsilon + \epsilon^2}{2}}\sigma_{TL}\cos\varphi,$$

has been taken into account. The Born terms and additional background were not included in this simulation.

For the purpose of the present study, both the scattered electron and proton had to be detectable in the trigger scintillation counters. This requirement provides very good momentum vector measurements for both particles. Additionally, we required that the outgoing electron is within the acceptance of the Cerenkov and shower counters. As a result, only a fraction of generated events are detectable in terms of experimental trigger requirement. In Fig. 1 we show the mass distribution of the generated (histogram) and accepted (crosses with error bars) events. Only the accepted events (henceforth called N_{obs}) by the CLAS detector will be used in the following analysis. In the following sections we will discuss the procedure to disentangle the various partial waves in the presence of configuration-dependent inefficiencies like acceptance losses in the CLAS detector.

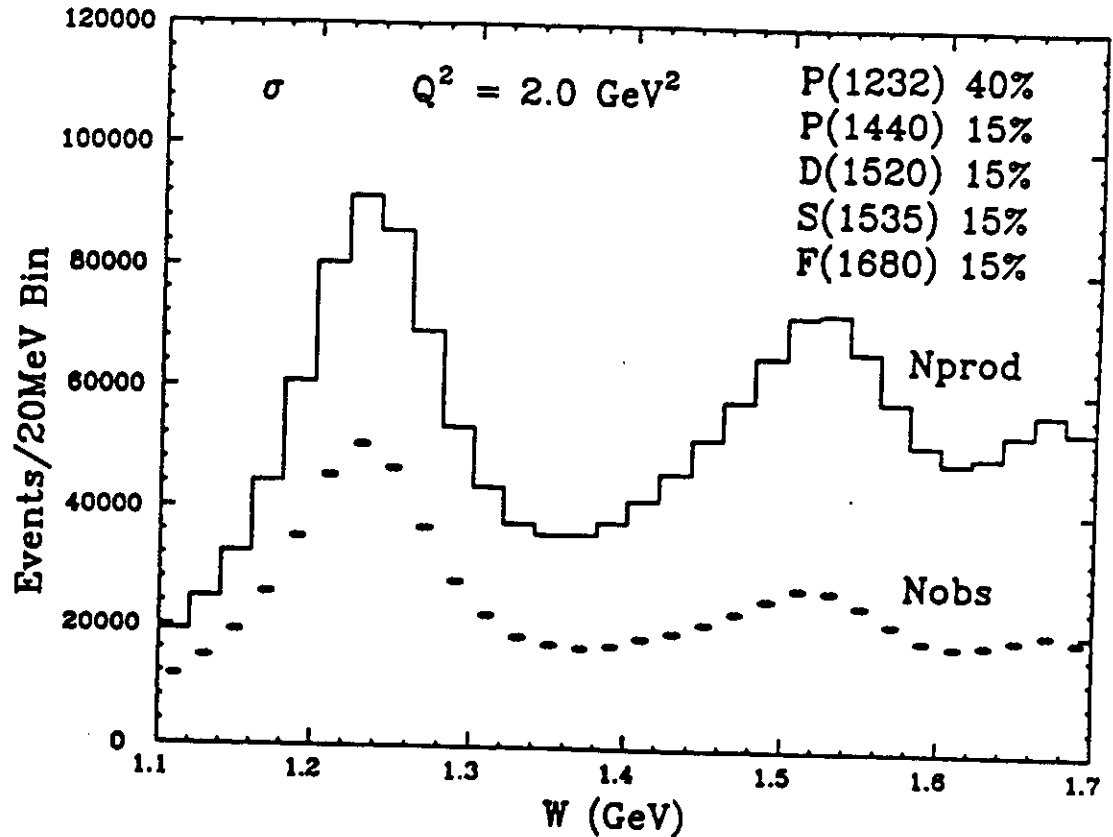


Fig.1. Invariant mass distribution of the generated N_{prod} (histogram) and accepted N_{obs} (crosses with error bars) events for the cross section σ .

II.2 Calculation of Acceptance

The average acceptance $A(\Delta Q^2, \Delta W, \Delta \cos\theta, \Delta\varphi)$ has been obtained by a Monte-Carlo calculation. For fixed values of $Q^2 = 2 \text{ GeV}^2$ and W in the range from 1.1 to 1.7 GeV, events were generated at random in the variables $\cos\theta, \varphi$ and transformed into the lab system with random rotation of the electron scattering

plane around the beam direction. The routines of FASTMC^[4] have been used to check if the scattered electron and final proton lie within the accepted geometry regions of the CLAS detector. Thus, for each bin $\Delta W = 20$ MeV, the average acceptance A has been determined in each bin of size $\Delta \cos\theta \Delta\varphi = (\frac{2}{40})(\frac{2\pi}{36})$, by generating 288,000 events (200 events/bin).

II.3 Fits to the $p\pi^0$ angular distribution

The angular distribution of the final hadron (π) is used to perform a model independent partial wave analysis in each $\Delta Q^2 \Delta W$ bin separately.

The procedure consists of fitting the observed event distribution N_{obs} (accepted by the CLAS) to the expected distribution in a grid of $(\cos\theta, \varphi)$ bins of equal size $\Delta\Omega$. Defining $w_k = N_{obs}$ as a total weight of observed events in bin k and w_k^{exp} as the expected total weight in the same bin, the χ^2 to be minimized can be written as

$$\chi^2 = \sum_{k=1}^{1440} \left[\frac{w_k - w_k^{exp}}{w_k^{exp}} \right]^2$$

with

$$w_k^{exp} = (\Delta\Omega/4\pi) A(\Omega_k) \sum_{\lambda=0}^{2l_{max}} \{ t_\lambda Y_\lambda(\Omega_k) + p_\lambda Y_\lambda(\Omega_k) \cos 2\varphi + r_\lambda Y_\lambda(\Omega_k) \cos \varphi \}$$

where $l_{max} = 3$ is the cut-off in the angular momentum (up to F-wave).

In the fits, $Y_\lambda(\Omega_k)$ is evaluated at the bin center whereas for $A(\Omega_k)$ the average acceptance in the bin is used. Bins with $A(\Omega_k) < 0.02$ have been excluded from the fit (for $W > 1.5$ GeV we found that only about 6% bins have the average acceptance $A(\Omega_k) < 0.02$). There are thus 21 parameters ($t_\lambda, p_\lambda, r_\lambda$) to be determined from the fit, in each $\Delta Q^2 \Delta W$ bin.

II.4 Discussion of the results

The mass dependence of $p\pi^0$ angular distribution moments t_L^M , p_L^M and r_L^M ($L_{max} = 6$ and $M = 0$), obtained from the fit as described above, are shown in Figs. 2,3 and 4. The moments t_0^0, p_0^0 and r_0^0 are the invariant mass spectra for σ , σ_{TT} and σ_{TL} correspondingly. For comparison, we also show the mass distribution of generated and accepted events. From this comparison one observes good agreement. All moments with odd L -values are consistent with zero, since in our model the Breit-Wigner forms were added incoherently. Generally, from the moments with odd L , one can learn about the interference between odd and even angular momentum l states. The moments with $M > 0$ (not included in our simulation) will show the contribution the states with angular momentum projection m different than zero.

The moments of angular distribution can be expressed in terms of the spin density matrix elements $\rho_{m_1 m_2}^{l_1 l_2}$. The relationship between the spherical harmonics moments t_L^0 (or p_L^0, r_L^0) and the intensities of partial waves S, P, D and F) as follows:

$$\begin{aligned} t_0^0 &= \rho_{00}^{00} + \rho_{00}^{11} + \rho_{00}^{22} + \rho_{00}^{33} = |S|^2 + |P|^2 + |D|^2 + |F|^2 \\ t_1^0 &= 0 \\ t_2^0 &= 0.894\rho_{00}^{11} + 0.639\rho_{00}^{22} + 0.596\rho_{00}^{33} = 0.894|P|^2 + 0.639|D|^2 + 0.596|F|^2 \\ t_3^0 &= 0 \\ t_4^0 &= 0.857\rho_{00}^{22} + 0.545\rho_{00}^{33} = 0.857|D|^2 + 0.545|F|^2 \\ t_5^0 &= 0 \\ t_6^0 &= 0.840\rho_{00}^{33} = 0.840|F|^2 \end{aligned}$$

where the explicit values are the Clebsch-Gordon coefficients.

Using the above relationships one can disentangle various partial waves. For example, in Fig. 5 we show the mass dependance of the partial waves (crosses) derived from the t_L^0 moments. For comparison we also show the generated mass distributions for the corresponding partial waves.

Having written down the relations between the moments and helicity amplitudes (will be discussed in a future paper), one may hope to find a unique solution for amplitudes and their W and Q^2 dependance.

Summary

It has been demonstrated that individual partial waves can be extracted from high statistics pion electroproduction data using the CLAS detector. The large acceptance of the CLAS spectrometer is a crucial factor for performing the proposed analysis. Such analysis is a necessary first step toward understanding electroproduced nucleon resonances.

Acknowledgment

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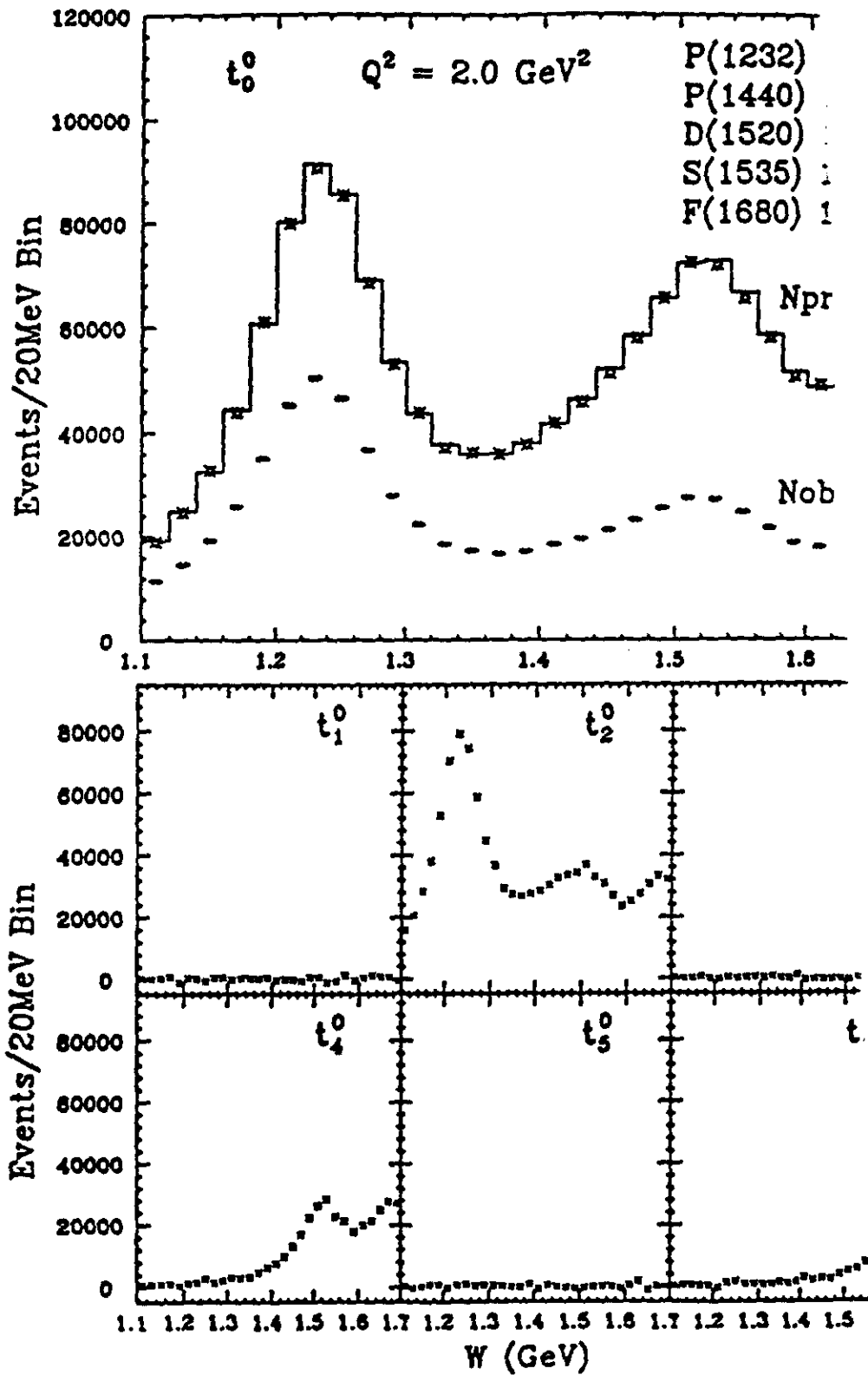


Fig.2. Mass dependence of $p\pi^0$ angular distribution moments t_L^M for the reaction σ (crosses without error bars). Histogram: produced (generated) events. Crosses with error bars: observed (accepted) events.

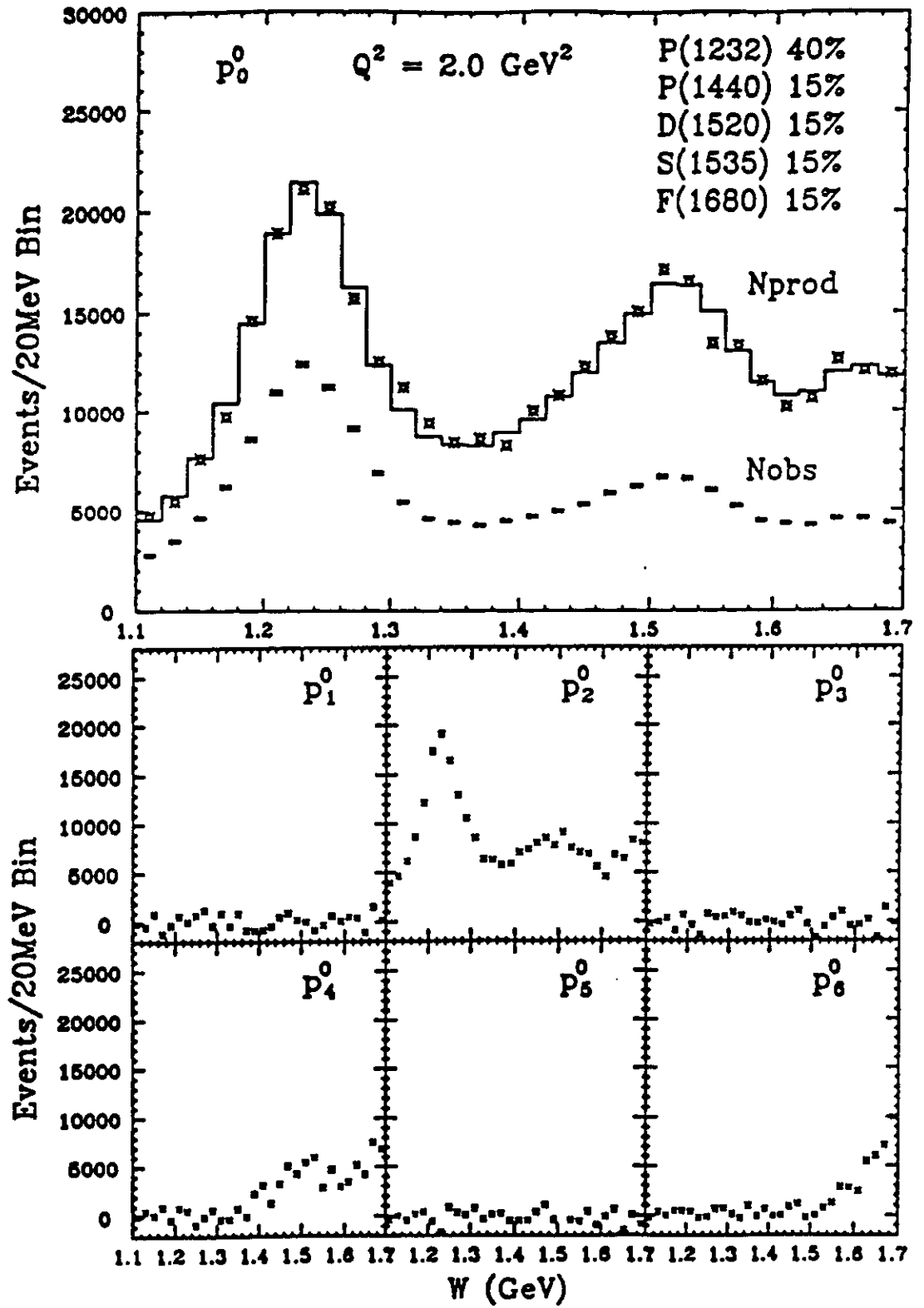


Fig.3. Mass dependance of $p\pi^0$ angular distribution moments p_L^M for σ_{TT} (crosses without error bars). Histogram: produced (generated) events. Crosses with error bars: observed (accepted) events.

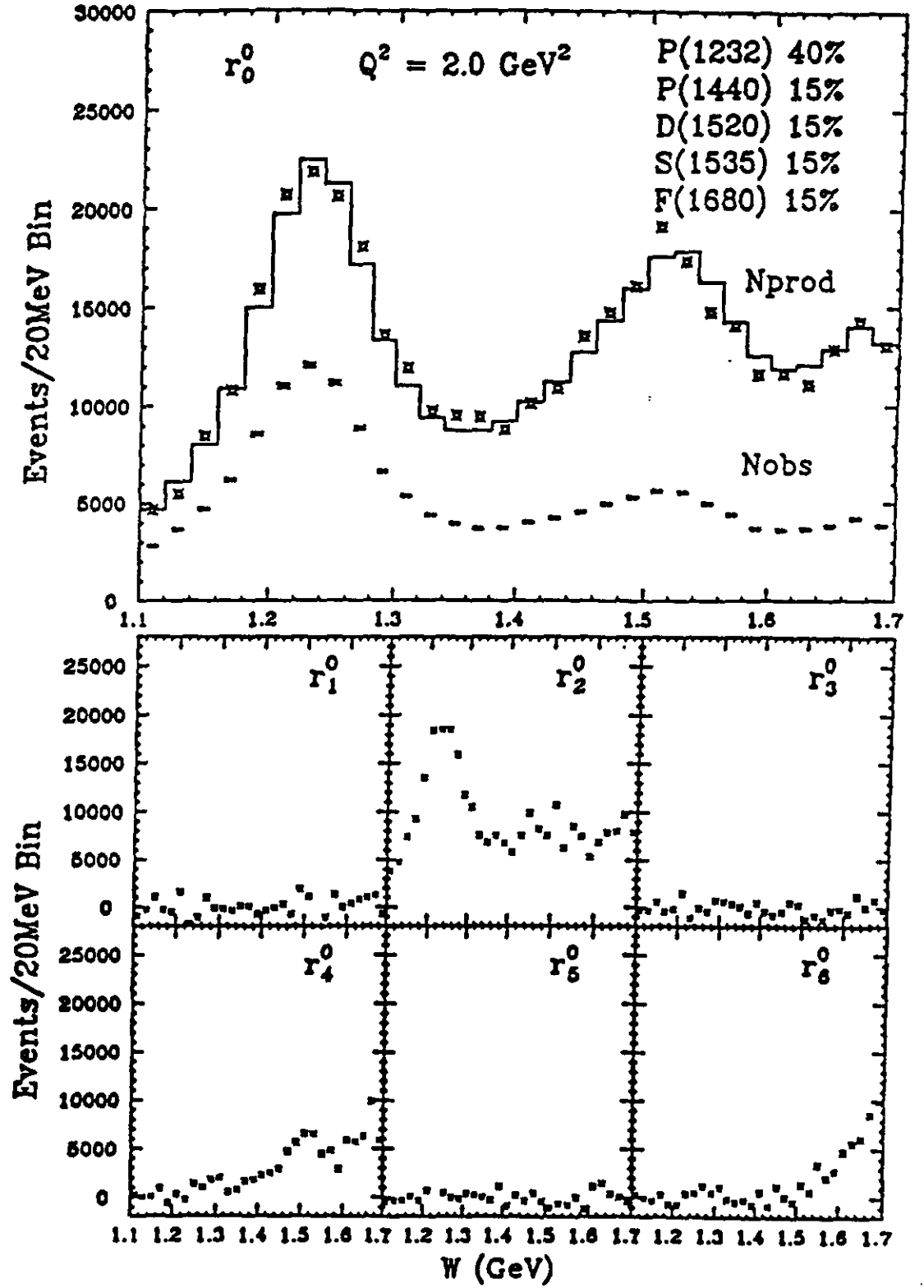


Fig.4. Mass dependence of $\pi\pi^0$ angular distribution moments r_L^M for σ_{TL} (crosses without error bars). Histogram: produced (generated) events. Crosses with error bars: observed (accepted) events.

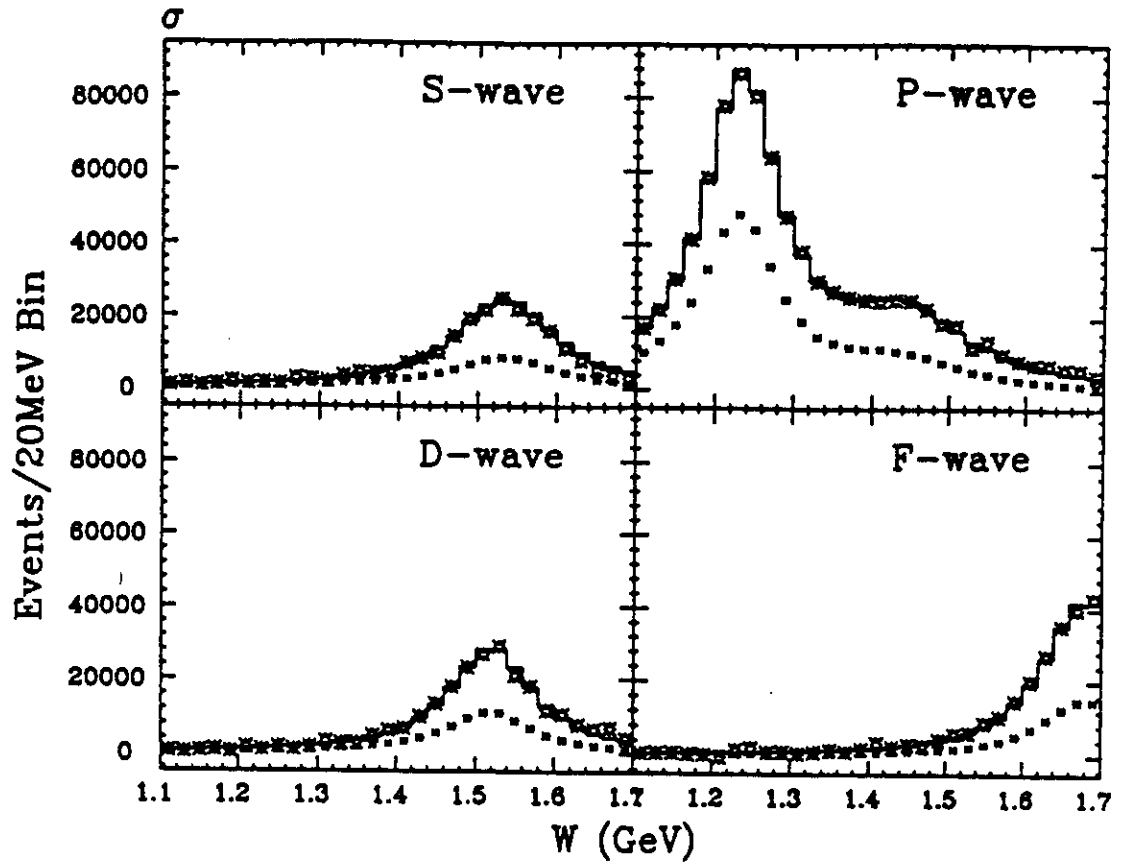


Fig.5. Mass dependance of the partial waves (crosses without error bars) derived from the t_L^M moments. Histogram: produced (generated) events. Crosses with error bars: observed (accepted) events.